AP Calculus
Properties of Limits

The following is a list of properties that can be applied to limits. NOTE: For any real number c, suppose the functions f and g both have limits at $x = c$

Constant Rule

$\lim_{x \to c} k = k$ for any constant k

Limit of x Rule

$$\lim_{x \to c}\left[s \cdot f(x)\right] = s \cdot \left[\lim_{x \to c} f(x)\right]$$

"The limit of a constant times a function is the constant times the limit of a function."

Example: $\lim_{x \to 0} 2 \cdot \sin(x) = 2 \cdot \lim_{x \to 0} \sin(x) = 2 \cdot 0 = 0$

Sum/Difference Rule

$$\lim_{x \to c}\left[f(x) \pm g(x)\right] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$$

"The limit of a sum is the sum of the limits"

Example: $\lim_{x \to c} f(x) = 7 \qquad \lim_{x \to c} g(x) = 4$

$$\lim_{x \to c}\left[f(x) + g(x)\right] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = 7 + 4 = 11$$

Product Rule

$$\lim_{x \to c}\left[f(x) \cdot g(x)\right] = \left[\lim_{x \to c} f(x)\right]\left[\lim_{x \to c} g(x)\right]$$

"The limit of a difference is the difference of the limits"

Quotient Rule

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} \quad \text{if} \quad \lim_{x\to c} g(x)\neq 0$$

"The limit of a quotient is the quotient of the limits, as long as the limit of the denominator is not zero."

Power Rule

$$\lim_{x\to c}\left[f(x)\right]^r=\left[\lim_{x\to c} f(x)\right]^n$$

Note: n is a rational number

"The limit of a power is the power of the limit."

Squeeze Rule

If on some interval around c

$$g(x)\le f(x)\le h(x) \qquad \text{and} \qquad \lim_{x\to c} g(x)=\lim_{x\to c} h(x)=L$$

Then

$$\lim_{x\to c} f(x)=L$$

"If a function can be squeezed between two functions with equal limits, then that function must also have the same limit."

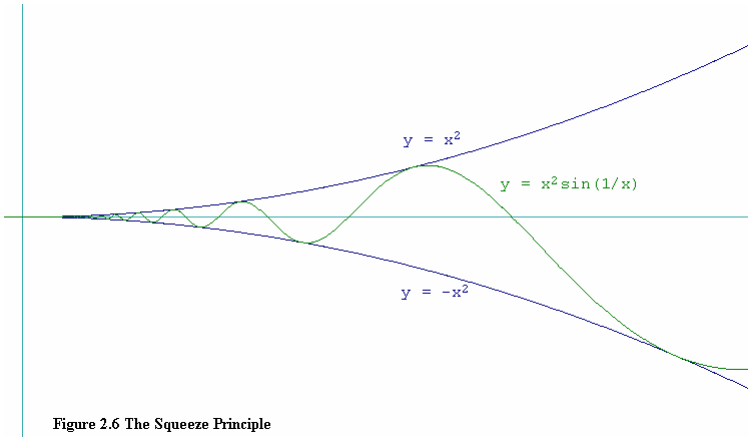

Figure 2.6 The Squeeze Principle